Coexistence of different vacua in the effective quantum field theory and Multiple Point Principle

G.E. Volovik
Low Temperature Laboratory, Helsinki University of Technology
P.O.Box 2200, FIN-02015 HUT, Finland
and
L.D. Landau Institute for Theoretical Physics RAS
Kosygina 2, 117940 Moscow, Russia

February 1, 2008

Abstract

According to the Multiple Point Principle our Universe is on the coexistence curve of two or more phases of the quantum vacuum. The coexistence of different quantum vacua can be regulated by the exchange of the global fermionic charges between the vacua, such as baryonic, leptonic or family charge. If the coexistence is regulated by the baryonic charge, all the coexisting vacua exhibit the baryonic asymmetry. Due to the exchange of the baryonic charge between the vacuum and matter which occurs above the electroweak transition, the baryonic asymmetry of the vacuum induces the baryonic asymmetry of matter in our Standard-Model phase of the quantum vacuum. The present baryonic asymmetry of the Universe indicates that the characteristic energy scale which regulates the equilibrium coexistence of different phases of quantum vacua is about 10^6 GeV.

1 Introduction

Dealing with quantum vacuum whose 'microscopic' physics is still unknown the high-energy, general-relativity and condensed-matter communities use different experience developed in working in each of those fields [1]. In condensed matter there is a rather general class of fermionic systems, where the relativistic quantum field theory gradually emerges at low energy and where the momentum-space topology is responsible for the mass protection for fermions, so that masses of all the fermions are much smaller than the natural energy scale provided by the microscopic (trans-Planckian) physics [2]. Since the vacuum of the Standard Model belongs to the same universality class of quantum vacua, this condensed-matter example provides us with some criteria for selection of the particle physics theories: the theory which incorporates the Standard Model must be consistent with its condensed-matter analog.

Here we apply such criteria to the Multiple Point Principle (MPP) [3, 4, 5, 6]. According to MPP, Nature chooses the parameters of the Standard Model such that two or several phases of the quantum vacua have the same energy density. These phases coexist in our Universe in the same manner as different phases of quantum liquids, such as superfluid phases A and B of ^{3}He or mixtures of ^{3}He and ^{4}He liquids. Using MPP Nielsen and co-workers arrived at some prediction for the correlation between the fine structure constants in their extension of the Standard Model. The fine tuning of the coupling constants is similar to the fine tuning of the chemical potentials of the coexisting quantum liquids in equilibrium.

The problem of the Standard Model parameters is thus related to the problem of the vacuum energy, and correspondingly to the cosmological constant problem. It was suggested [6] that MPP can serve as a basic principle to explain the present value of the cosmological constant. From the condensed-matter point of view such connection is rather natural. According to observations the cosmological constant is (approximately) zero in our phase of the quantum vacuum, that is why it must be (almost) zero in all the coexisting vacua as well. In condensed matter such nullification of the vacuum energy occurs for the arbitrary phase of the quantum vacuum. This happens due to the thermodynamic Gibbs-Duhem relation, according to which the microscopic (trans-Planckian) degrees of freedom exactly cancel the contribution to the vacuum energy from the low-energy (sub-Planckian) degrees of

freedom [2]. The phenomenon of nullification is so general that it must be applicable to any macroscopic system including the quantum vacuum of relativistic quantum fields, irrespective of whether the vacuum is true or false, and even if we do not know the microscopic physics.

Since the MPP is justified by condensed-matter analog, we can apply it to different problems related to quantum vacuum. Here we discuss the scenario of the baryonic asymmetry of the Universe, which follows from MPP.

2 Adjustment of the quantum vacuum after phase transition

The phase transitions between different quantum vacua does not influence the phenomenon of nullification of the vacuum energy: the energies of the vacuum is zero both above the transition and also after some transient period below the phase transition. In quantum liquids, the microscopic degrees of freedom which adjust themselves to nullify the energy density in a global equilibrium are the underlying bare particles – atoms of the liquid. The number density n of atoms changes after the phase transition and this compensates the change of the vacuum energy. For example, after the phase transition between superfluid ^{3}He-A and superfluid ^{3}He-B the relative change in the particle density is

$$\frac{\delta n}{n} \sim \frac{T_c^2}{E_F^2} \ . \tag{1}$$

Here the superfluid transition temperature T_c characterizes the energy scale of the superfluid phase transition, and also the transition between ^{3}He-A and ^{3}He-B; the Fermi energy $E_F \gg T_c$ characterizes the atomic (Planck) energy scale of the liquid. It is important that the correction to the microscopic parameter n (and also to E_F: $\delta E_F/E_F \sim \delta n/n$) is very small, and thus it does not influence the parameters of the effective theory of superfluidity in the low-energy corner.

The translation to the language of the Standard Model is almost straightforward. Let us consider the relative change of the microscopic (trans-Planckian) parameters needed to nullify the vacuum energy of the Standard Model after, say, the electroweak phase transition. In this case one must identify $T_c \equiv E_{\rm ew}$ and $E_F \equiv E_{\rm Pl}$ (the Planck energy). The rela-

tive change of the microscopic parameters in quantum liquids corresponds to the relative change of the Planck physics parameters, and thus one can identify $\delta n/n \equiv \delta E_{\rm Pl}/E_{\rm Pl}$. However, the equation (1) is not applicable for the Standard Model. The reason is that the fermionic density of states (DOS) in Standard Model above the electroweak phase transition differs from DOS in liquid ^{3}He above the superfluid phase transition. The vacuum in non-superfluid normal ^{3}He above the superfluid transition belongs to the Fermi-surface universality class, while the vacuum of the Standard Model above the electroweak transition belongs to the universality class with Fermi points. That is why they have different density of fermionic states: $N(E) \rightarrow {\rm Const} \sim E_F^2 \equiv E_{\rm Pl}^2$ in the vicinity of the Fermi surface and $N(E) \rightarrow E^2$ in the vicinity of the Fermi point. Thus the energy density related to superfluidity is $T_c^2 N(E = T_C) \sim T_c^2 E_{\rm Pl}^2$, while the energy denstity involved in the electroweak transition is $E_{\rm ew}^2 N(E = E_{\rm ew}) \sim E_{\rm ew}^4$. This gives an additional factor $E_{\rm ew}^2/E_{\rm Pl}^2$, as a result the relative correction to the Planck energy needed to compensate the energy change of the vacuum after the electroweak transition is

$$\frac{\delta E_{\rm Pl}}{E_{\rm Pl}} \sim \frac{E_{\rm ew}^4}{E_{\rm Pl}^4} \ . \tag{2}$$

Such response of the vacuum is so extremely small that it cannot influence the parameters of the effective low-energy theory – the Standard Model.

This demostrates that the adjustment of the deep vacuum does not lead to any sizable correlation between the parameters of the Standard Model, and thus the cosmological constant problem has nothing to do with the parameters of the effective theory. However, the MPP contains a more strong assumption than the statement that each vacuum always acquires zero energy. It assumes that several essentially different vacua have zero energy simultaneously, i.e. these vacua coexist in the same Universe (though the phase boundaries between different vacua can be well behind the cosmological horizon). The coexistence, though it does not influence the parameters of the effective theories, leads to other physical consequences, such as baryonic asymmetry of Universe. Let us discuss the principles of the coexistence of quantum vacua using as an example the coexisting quantum liquids, where the coexistence can be regulated both by microscopic and macroscopic parameters (analogs of microscopic or macroscopic fermionic charges).

3 Coexisting vacua

Let us first consider the quantum liquid formed by the mixture of k sorts of atoms. An example of the mixture of $k = 2$ components is provided by the liquid solution of ^{3}He atoms in ^{4}He liquid. The number of atoms N_a of each species a is conserved, and it serves as the conserved microscopic fermionic charge of the vacuum (the ground state of the mixture). The relevant vacuum energy whose gradient expansion gives rise to the effective quantum field theory for quasiparticles at low energy is [7]

$$\rho_{\rm vac} = \frac{1}{V}\left\langle \mathcal{H} - \sum_{a=1}^{k} \mu_a \mathcal{N}_a \right\rangle_{\rm vac} , \tag{3}$$

where $\mathcal{H}$ is the Hamiltonian of the system; $\mathcal{N}_a$ is the particle number operator for atoms of sort a in the mixture; and μ_a is their chemical potential. If the liquid is in equilibrium it obeys the Gibbs-Duhem relation which expresses the energy $E = \langle\mathcal{H}\rangle$ through the other thermodynamic varaibles including the temperature T, the entropy S, the particle number $N_a = \langle\mathcal{N}_a\rangle$ and the pressure P:

$$E - TS - \sum_{a=1}^{k} \mu_a N_a = -PV \ . \tag{4}$$

From this thermodynamic relation and Eq.(3) one obtains at $T = 0$ the familiar equation of state for the vacuum, which is valid both for relativistic and non-relativistic systems:

$$\rho_{\rm vac} = \epsilon - \sum_{a=1}^{k} \mu_a n_a \equiv \frac{1}{V}\left(E - \sum_{a=1}^{k} \mu_a N_a\right) = -P \ , \tag{5}$$

where we also introduced the energy density $\epsilon = E/V$ and particle number density $n_a = N_a/V$.

If the system is isolated from the environment, its pressure is zero and thus the energy density is zero too:

$$\rho_{\rm vac} = -P = 0 \ . \tag{6}$$

For such condensed-matter systems, in which the effective gravity emerges in the low-energy corner, this equation means that the effective cosmological

constant is zero. This nullification occurs without fine-tuning for any vacuum since the microscopic degrees of freedom – the particle number densities n_a and chemical potentails μ_a – automatically adjust themselves in equilibrium in such a way that the Gibbs-Duhem relation (4) is satisfied.

The more components the liquid has, the more flexible is the vacuum state, and as the result the number ν of different vacua which can coexist is bigger. In such flexible system the Multiple Point Principle naturally emerges. For the system with k components, the maximal number of different vacua which can coexist being separated by the phase boundaries is $\nu_{\rm max} = k$ (see Fig. 1 for $\nu = k = 3$), and all of these vacua have zero energy density: $\rho^{(i)}_{\rm vac} = 0$ ($i = 1, ..., \nu_{\rm max}$). This results from the following consideration. The coexisting vacua must have the same chemical potentials μ_a because of the exchange of particles between the vacua. They also have the same pressure $P = 0$ (and the same temperature $T = 0$). Thus for each vacuum i the pressure as a function of the chemical potentials must be zero: $P^{(i)}(\mu_1, \mu_2, ..., \mu_k) = 0$. All these ν equations can be satisfied simultaneously if $\nu \leq k$.

4 Coexistence of vacua regulated by effective fermionic charges

Does the coexistence of quantum vacua lead to observable consequences for the effective field theories emerging in these vacua? The answer is yes, if some of the variables n_a are soft variables belonging to the low-energy world, such as the density of the baryonic charge stored in the vacuum. An example is provided by the superfluid phases of ^{3}He, A and B, which can coexist at $T = 0$ and $P = 0$ in an applied magnetic field $\mathbf{H}$ [9]. The corresponding vacuum energy density is

$$\rho_{\rm vac} = \epsilon - \mu n - \mathbf{\Omega} \cdot \mathbf{S} \ , \tag{7}$$

where n is the number density of ^{3}He atoms; $\mathbf{S}$ is the density of the angular momentum which comes from the spins of the atoms (each atom has spin $\hbar/2$); $\mathbf{\Omega} = \gamma \mathbf{H}$; and γ is the gyromagnetic ratio of the ^{3}He atom. For a given direction of the magnetic field, say $\mathbf{H} = H\hat{\mathbf{z}}$, the liquid can be represented

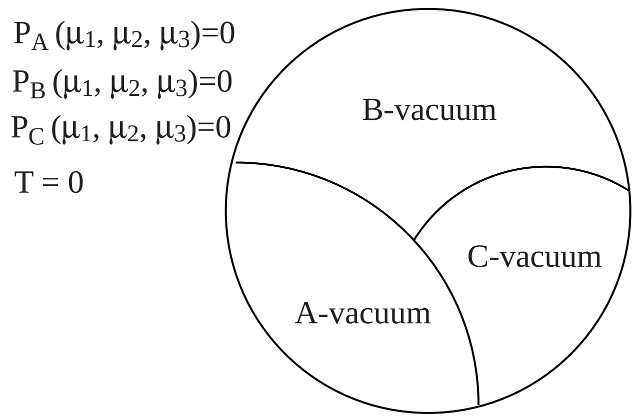

Figure 1: Example of $\nu = 3$ coexisting vacua A, B and C in a droplet of a substance with $k = 3$ conserved charges, N_1, N_2, and N_3. The droplet is isolated from environment, so that in all three vacua the pressure $P = 0$ if the curvature of the boundary of the droplet and the curvature of interfaces are neglected. That is why the energy density is also zero in all three coexisting vacua: $\rho_A = \rho_B = \rho_C = 0$. Volumes V_A, V_B and V_C occupied by the three vacua are determined by the total microscopic fermionic charges of the droplet: the particle numbers $N_1 = V_A n_{1A} + V_B n_{1B} + V_C n_{1C}$, $N_2 = V_A n_{2A} + V_B n_{2B} + V_C n_{2C}$ and $N_3 = V_A n_{3A} + V_B n_{3B} + V_C n_{3C}$.

as the mixture of the $k = 2$ components, with spin up and spin down:

$$\rho_{\rm vac} = \epsilon - \mu_\uparrow n_\uparrow - \mu_\downarrow n_\downarrow \ , \tag{8}$$

where

$$n_\uparrow = \frac{n}{2} + \frac{S_z}{\hbar} \ , \ n_\downarrow = \frac{n}{2} - \frac{S_z}{\hbar} \ , \ \mu_\uparrow = \mu + \frac{\hbar}{2}\Omega \ , \ \mu_\downarrow = \mu - \frac{\hbar}{2}\Omega \ . \tag{9}$$

Since we have effectively $k = 2$ components, the $\nu = 2$ vacua can coexist in the absence of the environment, i.e. at $P = 0$. This is the reason why A and B phases can coexist at $T = 0$ and $P = 0$.

As distinct from the variable n, the fermionic charge S_z is a soft variable since in typical situations it is zero (in the absence of external magnetic field). When the two vacua coexist, both variables n and S_z adjust themselves to nullify the pressure (and thus to nullify the 'cosmological constant' $\rho_{\rm vac}$) in each of the two phases. However, while the change of the particle density is negligibly small, $\delta n/n \propto T_c^2/E_F^2 \ll 1$, the change of the variable S_z is essential since it changes from zero. The energy density related to non-zero S_z is on the order of $S_z^2/E_{\rm Pl}^2$. Comparing this with the superfluid energy $T_c^2 E_{\rm Pl}^2$ one obtains that the characteristic spin density of the vacuum which emerges to compensate the AB phase transition is

$$|n_\uparrow - n_\downarrow| \sim T_c E_{\rm Pl}^2 \ , \ \frac{|n_\uparrow - n_\downarrow|}{|n_\uparrow + n_\downarrow|} \sim \frac{T_c}{E_{\rm Pl}} \ . \tag{10}$$

This is much bigger than the relative change in the particle density n after adjustment in Eq.(1). As a result the parameters of the effective theory, which describe superfluidity, also change considerably. In particular, the originally isotropic B-phase becomes highly anisotropic in the applied magnetic field (or at non-zero spin density of the liquid) needed for coexistence of A and B phases.

Note that S_z is the fermionic charge of the vacuum, and in principle it is not related to the fermionic charge of matter, since in our example the matter (quasiparticles) is absent. However, the charge asymmetry in the vacuum sector can cause the charge asymmetry in the matter sector due to exchange between the vacuum and matter. The resulting excess of the fermionic charge in the matter sector can induce the non-zero matter density even at $T = 0$. This is similar to the non-zero matter density in our Universe caused by the baryonic asymmetry of matter. Let us consider how this baryonic charge can be induced.

5 Coexistence and the baryonic asymmetry of the vacuum

Let us start with the baryonic charge of the vacuum of the relativistic quantum field exploiting an analogy between the macroscopic global charges: the spin S_z of the quantum liquid in its ground state and the global charges in our quantum vacuum, such as the baryonic charge B (or the family charge F [10]). The spin S_z of the liquid must be nonzero to provide the coexistence of the A and B vacua in superfluid ^{3}He at $T = 0$. In the same manner the baryonic charge B or family charge F could naturally arise in the quantum vacuum to establish the equilibrium between the coexisting phases of the vacuum.

Let us consider the coexistence of several vacua whose physics differ below the energy scale $E_{ce} \ll E_{\rm Pl}$. Such vacua can result from the broken symmetry phase transition, which occurs at $T \sim E_{ce}$, and we assume that the ordered phases differ by their residual symmetries H. The energy densities involved in the coexistence of the vacua are on the order of E_{ce}^4. Let us assume that the coexistence is regulated by the exchange of the baryonic charge between the vacua. Then one can estimate the density of this baryonic charge in the vacua by equating the energy density difference E_{ce}^4 and the energy density of the vacuum due to the nonzero charge density B. If the baryonic charge is stored in the microscopic degrees of freedom of the quantum vacuum, the energy density related to this charge must be on the order of $B^2/E_{\rm Pl}^2$. As a result the baryonic charge density of the vacuum needed for the coexistence of different vacua is

$$B_{\rm vac} \sim E_{ce}^2 E_{\rm Pl} \ . \tag{11}$$

Thus the coexistence results in the baryonic asymmetry in the vacuum sector. In turn, the non-zero baryonic charge of the vacuum could be in the origin of the baryonic asymmetry of the matter in our Universe – an excess of the baryons over antibaryons, $n_B > n_{\bar{B}}$. Let us consider this mechanism of baryogenesis.

6 From baryonic asymmetry of the vacuum to baryonic asymmetry of Universe

If an exchange of the baryonic charge between the vacuum and matter is possible, the chemical potential for the baryons in matter must be equal to the chemical potential for the baryonic charge in the vacuum. The latter is non-zero due to the non-zero baryonic charge in the vacuum sector in Eq.(11):

$$\mu_B \sim \frac{B_{\rm vac}}{E_{\rm Pl}^2} \sim \frac{E_{ce}^2}{E_{\rm Pl}} ~. \tag{12}$$

At temperature $T \gg \mu_B$ one obtains the following estimation for the baryonic charge stored in the matter sector (in the gas of relativistic fermions):

$$B_{\rm matter} = n_B - n_{\bar B} \sim T^2\mu_B \sim \frac{T^2 E_{ce}^2}{E_{\rm Pl}} ~. \tag{13}$$

However, the exchange between the vacuum and matter occurs (due to axial anomaly) only at T above the electroweak transition, $T > E_{\rm ew}$. Below the transition, at $T < E_{\rm ew}$ the exchange with the quantum vacuum is highly suppressed: the transition rate due to the sphaleron mechanism becomes exponentially weak [11, 12]. At the moment of the phase transition, i.e. at $T \sim E_{\rm ew}$, the baryonic asymmetry of matter (primordial baryon-to-entropy ratio) is:

$$\eta = \frac{n_B - n_{\bar B}}{s} \sim \frac{T^2\mu_B}{T^3} \sim \frac{E_{ce}^2}{E_{\rm ew}E_{\rm Pl}} ~. \tag{14}$$

Below the transition, the baryonic charge in the matter sector is completely separated from the vacuum and evolves together with matter, while the density of the baryionic charge in the vacuum sector remains constant.

In the matter sector the baryonic density evolves in the same way as the entropy, and thus the baryon-to-entropy ratio η remains the same as at the moment of transition. To obtain the value $\eta \sim 10^{-10}$, which follows from the cosmological observations [11, 12], the characteristic energy E_{ce}, related to the baryonic charge of the vacuum, must be

$$E_{ce} \sim 10^{-5}\sqrt{E_{\rm ew}E_{\rm Pl}} \sim 10^6 ~{\rm GeV} ~. \tag{15}$$

In analogy with A and B phases of ^{3}He, this corresponds to the transition temperature T_c at which the coexisting vacua were formed due to symmetry breaking.

The main point in this scenario of the baryogenesis is that the vacuum and matter are two subsystems, whose properties related to the fermionic (baryonic) charge are different. In condensed matter the analogous exchange of spin charge between the superfluid vacuum and quasiparticles (matter) plays an important role in the spin dynamics of the system (see [13] and Sec.8.6 in [9]).

7 Conclusion

In conclusion, the gravitating part of the vacuum energy is always zero in equilibrium vacuum, $\rho_{\rm vac} = 0$, even if the cosmological phase transition occurs. The nullification after the phase transition is supported by automatic adjustment of the microscopic ultraviolet degrees of freedom. However, because of the huge energy stored in the microscopic degrees, the relative change in the microscopic parameters is extremely small, and this adjustment practically does not influence the parameters of the effective infrared theories. As a result, the Multiple Point Principle, which implies the coexistence of two or several different (i.e. not connected by symmetry) naturally occurs, and all the coexisting vacua automatically acquire zero energy without any fine-tuning.

If the Universe is on the coexistence curve, this may lead to the observable physical consequences related to the fermionic charges of the vacuum and matter. In particular, if the coexistence is regulated by the exchange of the baryonic charge, all the coexisting vacua acquire the baryonic asymmetry. The latter in turn gives rise to the baryonic asymetry in the matter sector.

According to Eqs.(15) and (11) the density of the baryonic charge in the vacuum sector is rather high, $B_{\rm vac} \sim 10^{-26} E_{\rm Pl}^3$. What are the consequences of such CP violation in the quantum vacuum is the subject of further investigations.

I thank A.F. Andreev for illuminating discussion. This work was supported by ESF COSLAB Programme and by the Russian Foundations for Fundamental Research.

References

[1] Giovanni Amelino-Camelia, The three perspectives on the quantum-gravity problem and their implications for the fate of Lorentz symmetry, gr-qc/0309054.

[2] G. E. Volovik, *The Universe in a Helium Droplet*, Clarendon Press, Oxford (2003).

[3] D.L. Bennett, C.D. Froggatt and H.B. Nielsen, in: Proceedings of the 27th International Conference on High Energy Physics, eds. P. Bussey and I. Knowles, IOP Publishing Ltd, 1995, p. 557.

[4] C.D. Froggatt, The problem of mass, hep- ph/0312220.

[5] C.D. Froggatt and H.B. Nielsen, Trying to understand the Standard Model parameters, hep-ph/0308144.

[6] C. Froggatt, L. Laperashvili, R. Nevzorov and H.B. Nielsen, Cosmological constant in SUGRA models and the multiple point principle, hep-ph/0310127.

[7] A.A. Abrikosov, L.P. Gorkov and I.E. Dzyaloshinskii, *Quantum Field Theoretical Methods in Statistical Physics*, Pergamon, Oxford (1965).

[8] E.M. Lifshitz and L.P. Pitaevskii, Statistical Physics, Pergamon Press, 1980.

[9] D. Vollhardt and P. Wölfle, *The superfluid phases of helium 3*, Taylor and Francis, London (1990).

[10] G.E. Volovik, Dark matter from $SU(4)$ model, JETP Lett. **78**, 691–694 (2003).

[11] M. Dine and A. Kusenko, Rev. Mod. Phys. **76**, 1 (2004).

[12] M. Trodden, Electroweak baryogenesis, Rev. Mod. Phys. **71**, 1463–1499 (1999).

[13] A.J. Leggett and S. Takagi, NMR in A-^{3}He and B-^{3}He: The intrinsic relaxation mechanism, Phys. Rev. Lett. **34**, 1424–1427 (1975); Ann. Phys. **106**, 79 (1977).